A flexible and path-preserving algorithm for curvilinear P^2 mesh generation based on Riemann geometry

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Abstract

We propose a novel framework for curvilinear mesh generation and adaptation in two dimensions that consists of several key components: A new algorithm for generating boundary mesh points ensures a uniform distribution along each boundary line. A flexible algorithm for generating mesh points in the computational domain allows meshing in optional numbers of directions or combinations thereof. A constrained Delaunay triangulation method that preserves the vertex generation paths. An enhanced algorithm for curving straight-sided edges improves the accuracy and efficiency of curvilinear edge triangles. Improvements in both the straight-sided edge swapping and curvilinear edge swapping algorithms increase the overall efficiency.

We present a detailed description of the mesh generation algorithms. We explain how points are sampled both at the boundary of the domain and within the domain to ensure an optimal distribution. The points are then triangulated using a novel constrained Delaunay triangulation method to ensure that the initial mesh is well-posed and respects the constraints of the problem. We also include a discussion of curving the mesh along geodesic paths and introduce an improved backtracking algorithm with constrained optimization. This algorithm minimizes the length of the geodesic parabola while ensuring that neighboring triangles remain valid, thus maintaining geometric consistency throughout the refinement process. In addition, we introduce some improvements to reduce the computational cost of straight-sided edge swapping and curvilinear edge swapping techniques, which align the mesh with the desired anisotropic metric field and allow the mesh to adapt to the specific needs of the simulation.

We present several numerical examples to demonstrate the effectiveness of our mesh adaptation framework. These examples include simple test cases that highlight the application of the curvilinear mesh and the impact of high-order adaptation on solution accuracy. The results show the potential of this approach for solving complex physical problems where both high geometric fidelity and high numerical accuracy are essential.

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