Surface and Hypersurface Meshing Techniques for Space-Time Finite Element Methods

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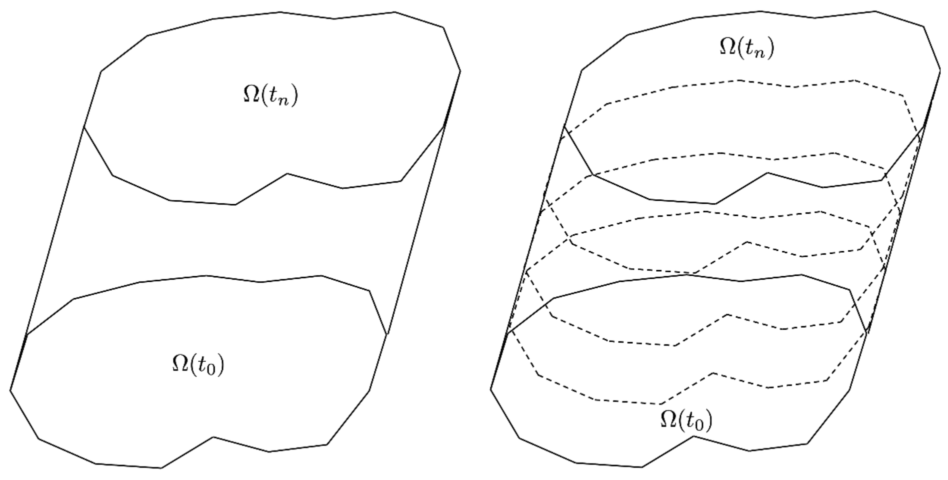
Abstract

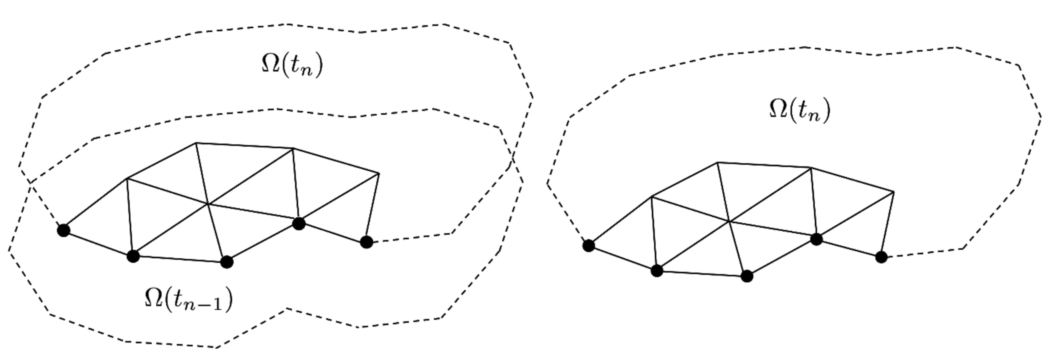
A general method is introduced for constructing two-dimensional (2D) surface meshes embedded in three-dimensional (3D) space time, and 3D hypersurface meshes embedded in four-dimensional (4D) space time. In particular, we begin by dividing the space-time domain into time slabs. Each time slab is equipped with an initial plane (hyperplane), in conjunction with an unstructured simplicial surface (hypersurface) mesh that covers the initial plane. We then obtain the vertices of the terminating plane (hyperplane) of the time slab from the vertices of the initial plane using a space-time trajectory-tracking approach. Next, these vertices are used to create an unstructured simplicial mesh on the terminating plane (hyperplane). Thereafter, the initial and terminating boundary vertices are stitched together to form simplicial meshes on the intermediate surfaces or sides of the time slab.

Preliminaries

We begin by partitioning the space-time domain into slabs. In 3D, each slab contains an initial plane (at $t = t_n$), a terminating plane (at $t = t_{n+1}$), and an intermediate surface which connects the initial plane to the terminating plane. Space-time slabs are formed sequentially as needed with information from the previous slab's terminating plane forming the new slab's initial plane.



Partitioning space-time domain into slabs



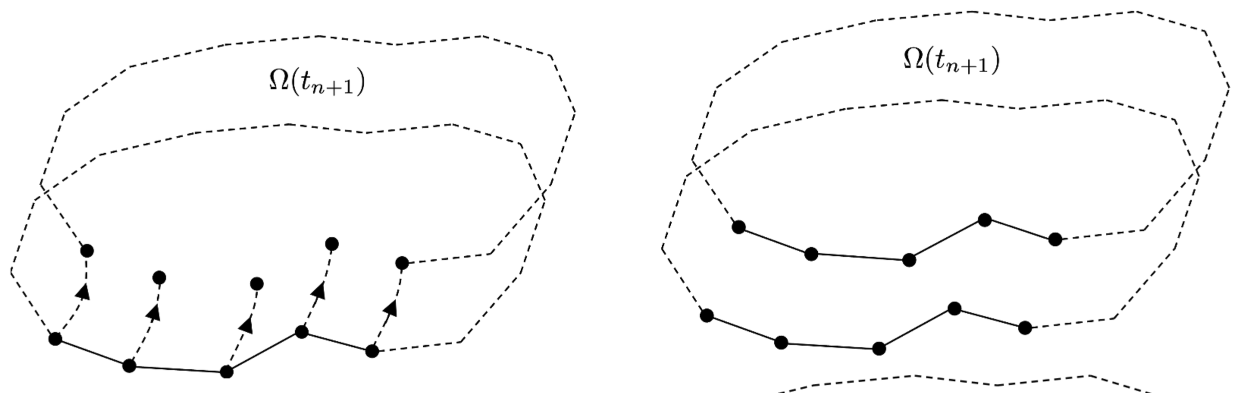
Forming the initial plane of a new time slab

Vertex Trajectory-Tracking and Quad Splitting 2D

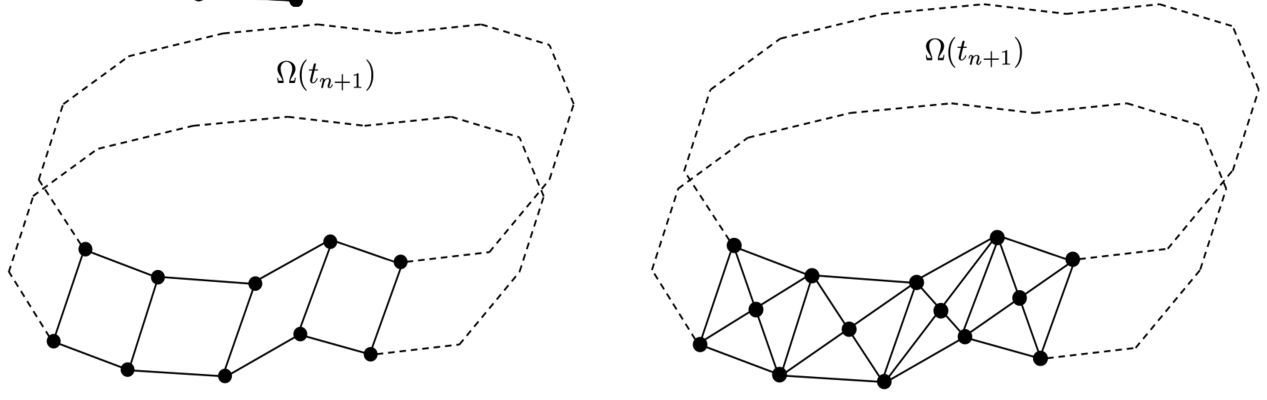
Extract the boundary edges and vertices of the surface mesh from the initial plane. Compute the vertex trajectories from $t = t_n$ to $t = t_{n+1}$. Project the final point locations to the CAD surface. Connect the vertices on the terminating plane to create edges. Connect edges on the terminating plane to edges on the initial plane to create quadrilaterals. Subdivide the quadrilaterals into triangles to generate a triangular surface mesh on the intermediate surface. Use the edges on the terminating plane to generate a triangular surface mesh on the terminating plane.

Vertex tracker: $v(t) = \frac{dx(t)}{dt}$

Tracking vertices of initial plane to form edges on terminating plane



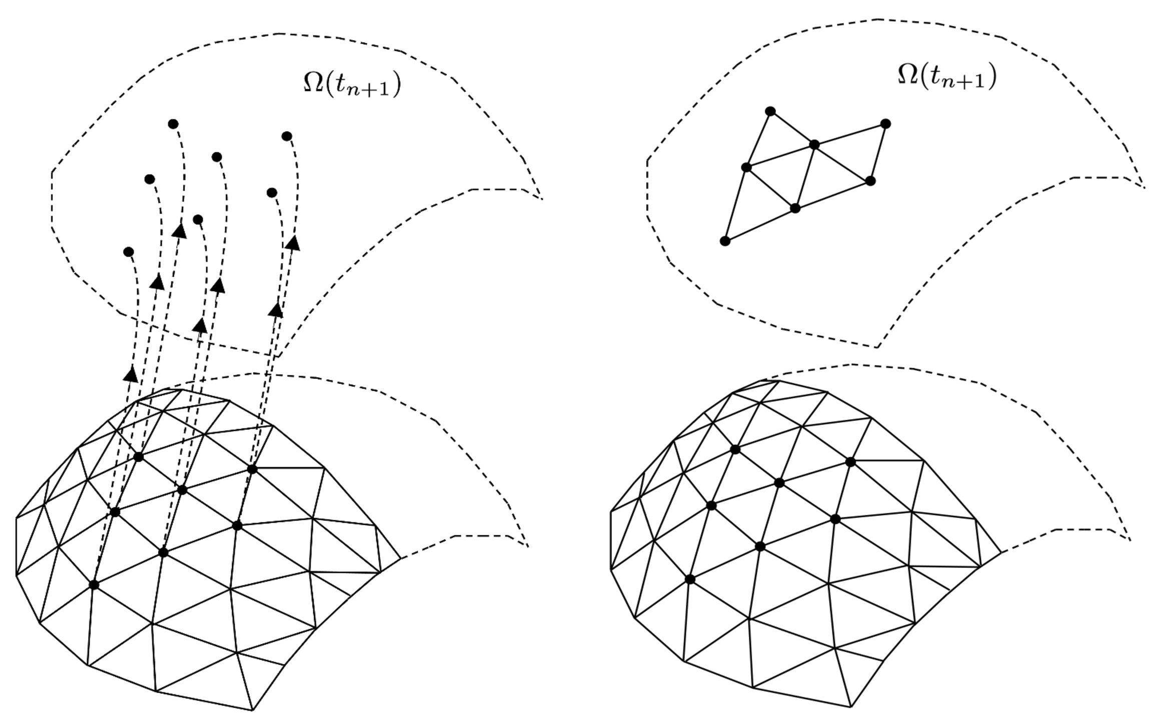
Vertices are stitched together to form quads that are split into triangles



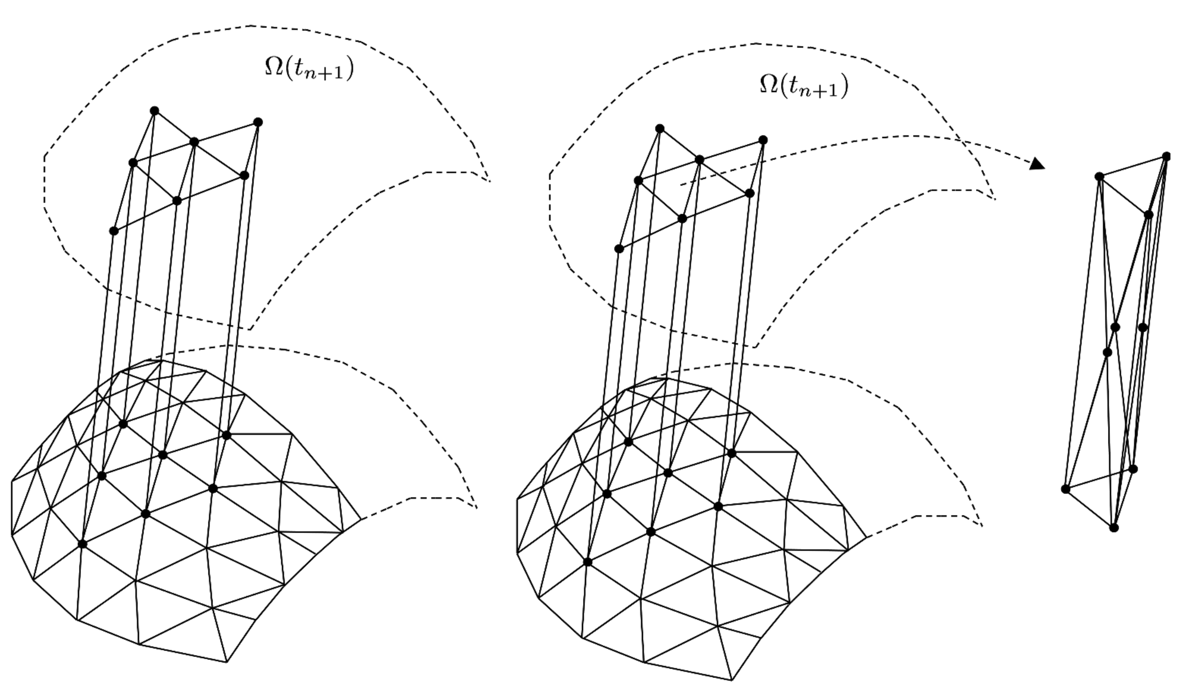
Vertex Trajectory-Tracking and Prism Splitting 3D

Extract the boundary triangular faces, edges, and vertices of the hypersurface mesh from the initial plane. Compute the vertex trajectories from $t = t_n$ to $t = t_{n+1}$. Project the final point locations to the CAD surface. Connect the vertices on the terminating hyperplane to create triangular faces. Connect the triangles on the terminating hyperplane to triangles on the initial hyperplane to create triangular prisms. Subdivide the triangular prisms into tetrahedra to generate a tetrahedral hypersurface mesh on the intermediate hypersurface. (There are multiple schemes for splitting a triangular prism into tetrahedra. We found options C and E below to be most suitable.) Use the triangular faces on the terminating hyperplane to generate a tetrahedral hypersurface mesh on the terminating hyperplane.

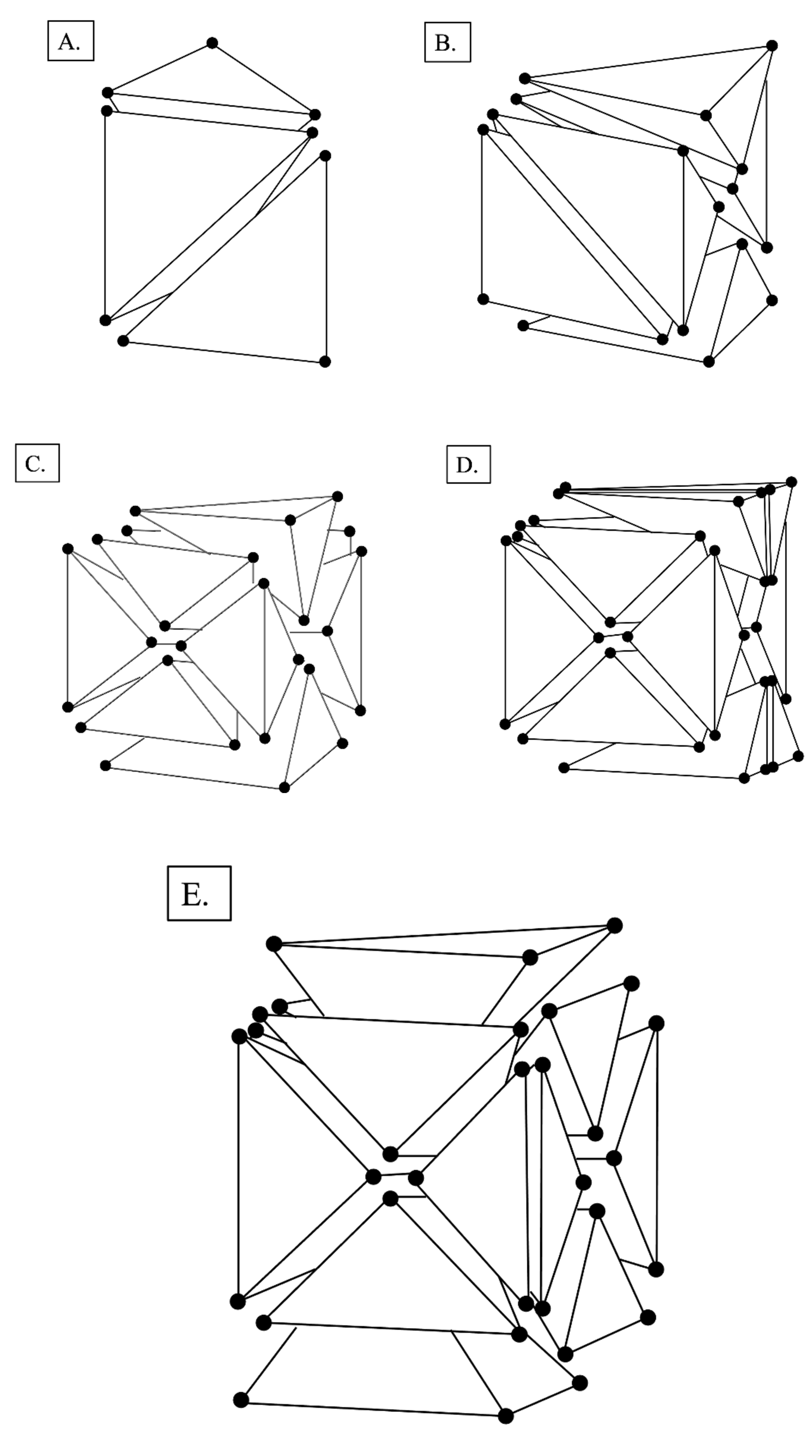
Tracking vertices of initial hyperplane to form triangles on terminating hyperplane



Vertices are stitched together to form triangular prisms that are split into tetrahedra

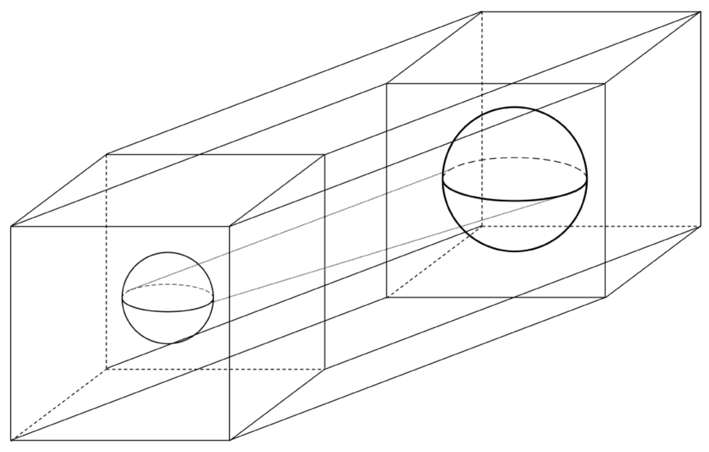


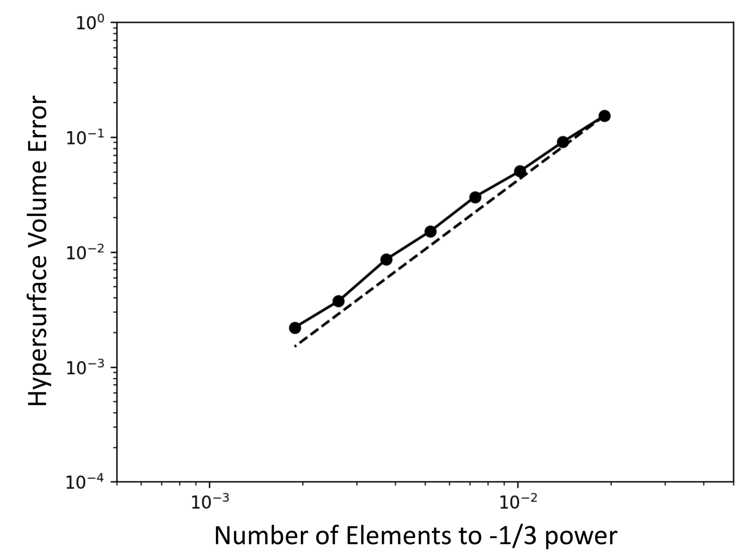
Schemes for splitting a triangular prism into tetrahedra.



Results: Expanding Sphere

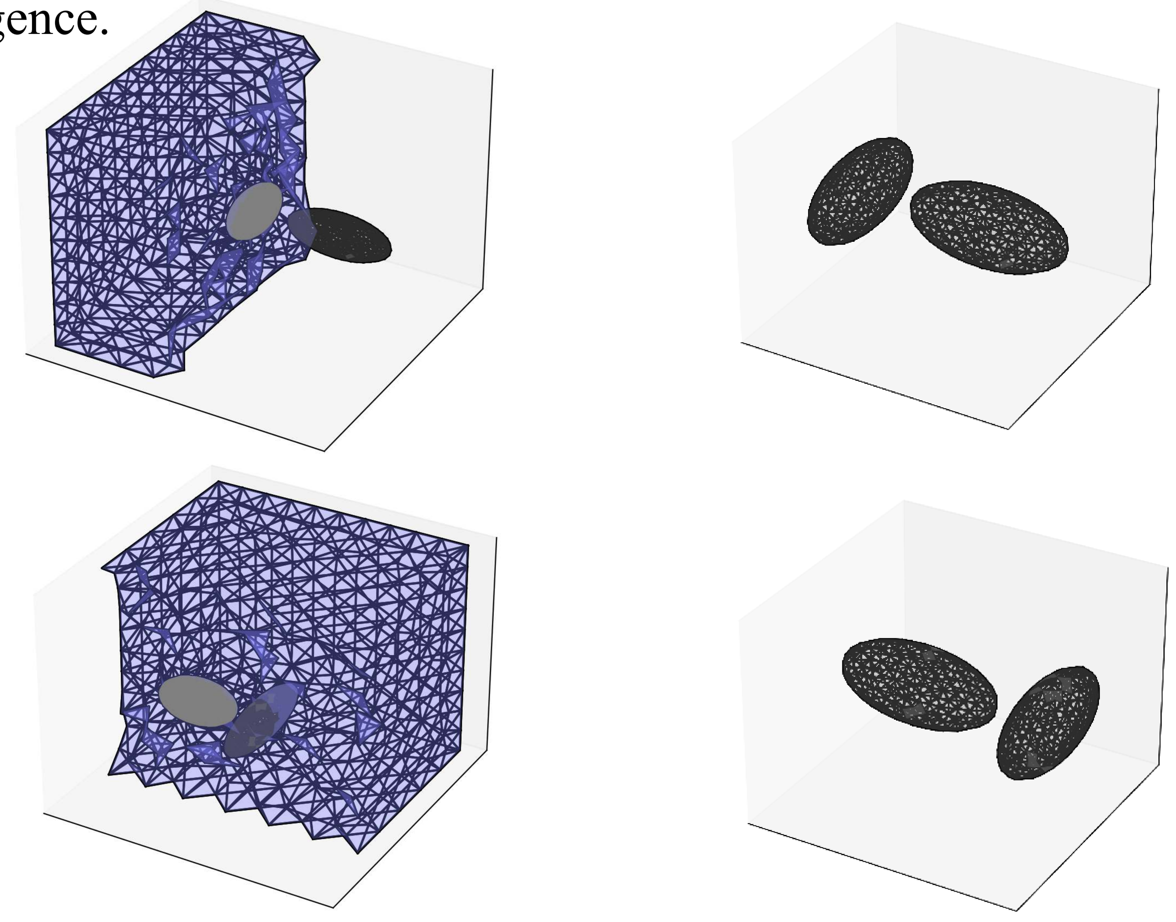
In this experiment, a stationary sphere is expanding from $R_0 = 1$ to $R_f = 1.25$ over the time interval [0, 1]. The associated space-time geometry consisted of a hyper-conical frustum embedded inside of a tesseract. The total volume of each mesh was compared with the exact volume of the slab's hypersurface. As shown in the figure below, the error in the approximation deceases with increasing mesh resolution, achieving second-order convergence.

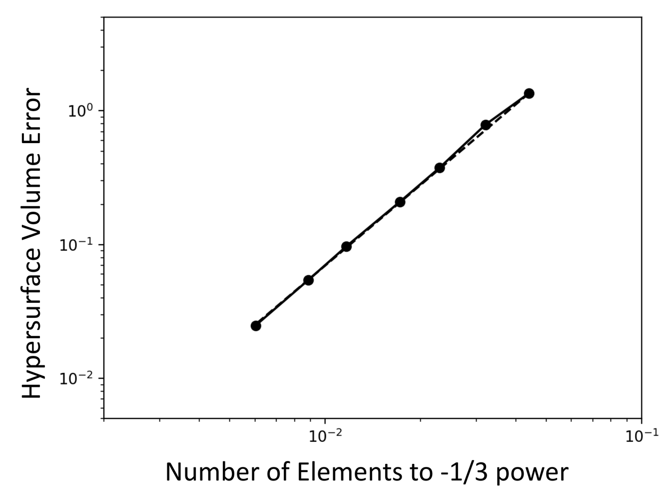




Results: Rotating Tandem Ellipsoids

For this experiment, the geometry consisted of a pair of ellipsoids with semi-axes of $a_1 = 1$, $b_1 = 3$, $c_1 = 2$ and $a_2 = 3$, $b_2 = 1$, $c_2 = 2$, respectively. The first ellipsoid rotated with angular velocity $(0, 0, \pi/2)$ rads/s, and the second rotated with velocity $(0, 0, -\pi/2)$ rad/s. On the time interval [0, 1], the motion of the ellipsoids created a pair of elliptical hyper-helixes that were contained inside of a tesseract. The figure below shows a plot of the volumetric error versus the mesh resolution; we achieve second-order convergence.





Acknowledgements

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